Is 'Capacity' Volume? Understandings of 11 to 12-year-old Children

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This paper reports a study that explored 11 to 12-year-olds' conceptions of the terms 'volume', 'capacity' and 'interior volume'. Semi-structured interviews, supported by the use of measurement resources, were used to gain insight into their thinking. Video-recordings of the interviews allowed analysis of dialogue, actions with the materials and gestures used by the students. In general, the students believed that 'capacity' was different to 'volume' but were confused about the difference. The term 'interior volume' was easily understood by the students to mean the space inside a hollow container. We argue that the ambiguity of the term 'capacity' is reflected in the children's confusion about what is actually being measured and the method that should be used to measure it.

While there is a considerable body of mathematics education literature on the development of children's conceptions of the spatial structure of units of volume, there is little research on how it is integrated with their understanding of capacity, and even less on the efficacy of their use of the terms 'volume' and 'capacity'. The correct usage of mathematical language is important to students' conceptual understandings, so Lowrie, Logan, and Scriven (2012) recommend that teachers use precise mathematical language with students from an early age. It is also acknowledged that ambiguity of language can form a barrier to understanding mathematical concepts (COAG, 2008), and that words with different meanings beyond school can cause confusion when introduced in mathematics classes (Meiers & Trevitt, 2010). The terms 'volume' and 'capacity' both have more than one meaning. 'Capacity' is especially problematic because it has a similar but broader meaning outside school mathematics. It is concerning that school curricular present teachers and children with a variety of interpretations for these terms, a somewhat unusual occurrence in mathematics.

Meanings for 'Volume' and 'Capacity'

It is debatable whether 'capacity' is a mathematical term. In everyday contexts, the capacity of a container could be the number of items a container will hold (e.g., the number of lollies a lolly jar will hold) or the mass a container will hold (e.g., a 250-gram jam jar) and it does not have to be a container to have capacity. The Singapore Mathematics Curriculum (Singapore Ministry of Education, 2012) uses the word 'capacity' in the outcome, "Estimate a big number (e.g. the seating capacity of the Singapore Indoor Stadium)" (p. 47). When referring to capacity, this curriculum clarifies what is meant by writing it as 'volume/capacity', for example, "collect familiar objects with varying volume/capacity, e.g., cough syrup spoons, syrup bottles, food containers" (p.44).

The Australian Curriculum: Mathematics glossary states that capacity is "a term that describes how much a container will hold. It is used in reference to the volume of fluids or gases and is measured in units such as litres or millilitres" (ACARA, 2017). The words 'how much a container will hold' can be misunderstood. Asking which container 'will hold more' when containers are partially filled, is ambiguous as it could be understood as how much space is left to fill, or the fullness of the container (Harrison, 1987).

Unlike the Australian definition, the glossary of New Zealand Mathematics curriculum

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does not restrict the units of capacity to liquid units such as litres or millilitres. It defines capacity as "a measure of the interior volume of a container. Hence it is a measure of how much a container can hold. It is measured in units of volume." (New Zealand Government, 2019). The term 'interior volume' was the term used by Piaget to mean the volume of a space defined by a physical boundary (Potari & Spiliotopoulou, 1996). Other early researchers who based their work on Piagetian tasks also used the term 'interior volume' or 'internal volume' (Feghali, 1979). However, their research did not explore whether using these terms would support clarity of meaning for children.

Previous research has emphasised the need for children to conceptualise the measurement attributes before they are taught how to measure them (Passelaigue & Munier, 2015), and this is reflected in the sequence of outcomes in the Australian Curriculum: Mathematics (ACARA, 2017). Notably, 'volume' and 'capacity' are dealt with as separate attributes. In the Foundation Year, students are expected to make direct comparisons by pouring from one container to another to reason which "holds more". Subsequent outcomes within the curriculum treat 'capacity' as an attribute separate from volume, and different units are used. Volume and capacity are not related to each other until Year 6 with the outcome: "Connect volume and capacity and their units of measurement" (ACMMG138), including recognising that 1mL is equivalent to 1cm^3.

The New South Wales Department of Education (NSWDE, 2018, p.76) considers 'capacity with liquid units' to be one of five different aspects of volume, the other four aspects being 'volume presented as a model made with blocks'; 'interior volume' (meaning the volume inside a container, measured in cubic units); 'exterior volume' (meaning the amount of space a container 'takes up' i.e., the material volume of the container together with its capacity); and 'volume measured as displacement'.

Linking the Meaning of the Attribute with the Method of Measuring

Some of the five 'aspects' listed in the paragraph above, are better described as 'types' of volume and others as 'processes' for measuring volume. The exterior volume of a closed container, the occupied volume of an open container and the volume of a solid could all be measured by displacement. 'Interior volume' and 'capacity' are considered different 'aspects' yet both relate to filling a container to find how much it can hold.

Prior research on the measurement of volume, has primarily focused on children's understandings of volume measured by stacking cubes or packing them into rectilinear containers and counting them in rows and layers (Battista, 2003; Curry & Outhred, 2005), but has not related this to their understanding of volume measured by 'filling' a container. Abstracting volume requires conceptualisations of deformable volume units, such as play-dough, that maintain their volumes when re-shaped, allowing students to progress from seeing volume as a number of rigid cubes within a region (Battista, 2003). The linking of different methods and units for measuring (e.g., cubes vs water and the equivalence of 1mL and 1cm^3) appears to be an important step in understanding volume/capacity, and may influence the interpretation of the specific terms.

Research Question

The inconsistency of meanings for 'volume' and 'capacity' in various curriculum documents, demonstrates the potential for confusion in the development of understanding these attributes, and the relationship between them. As highlighted by Battista (2003), discovering students' existing conceptions of measurement attributes can enable educators to teach measurement in a manner that stimulates meaningful learning of the underlying ideas. Therefore, this exploratory study is focused by the question: *What are 11 to 12-*

year-old children's conceptions of the terms 'volume', 'capacity' and 'interior volume'?

Methods

The methodology of this study was predominantly influenced by the studies of Curry and Outhred (2005) who interviewed students about their understanding of spatial measurement. It was also influenced by Potari and Spiliotopoulou (1996) and Sisman and Aksu (2016) who, through written tasks about volume given to fifth and sixth grade children respectively, found a lack of comprehension of fundamental concepts regarding 3D space. Semi-structured interviews provide opportunities for children to express, discuss and clarify their ideas and thought-processes as opposed to written and drawn responses to test-style questions (Sisman & Aksu, 2016).

The Participants

Six children participated in the study: Alex, Billy, Chris, Drew, Emmet and Fargo (pseudonyms). They were recruited by distributing leaflets to people with children aged 11 or 12. The children attended six different schools across suburbs of Sydney. All were at public schools, except Alex who attended an independent school. The schools' ICSEA values ranged from 898 (Chris) to 1170 (Drew). The interviews were conducted midway in the year, when five of the children were in Year 6 and one child (Drew) was in Year 7.

The Semi-structured Interviews

Semi-structured interviews with the children were conducted by the first author while a second researcher took field notes, and the interviews were video-recorded. The interviews were semi-structured to enable clarification of questions and responses through the use of gesturing, which is critical for communicating spatial information (Ehrlich, Levine, & Goldin-Meadow, 2006). The length of the interviews varied from eleven to twenty-eight minutes depending on the individual's responses and their willingness to continue the discussion.

The focus was not on quantitative characteristics of volume such as numerical values and the use of formulas, but rather on the child's conception of the attributes of volume and capacity. The interviewer sought to discover: (a) what 3D space they understood 'volume' to mean when presented with a container and how they chose to measure this space; (b) what they understood 'capacity' to mean and how they chose to measure it; and, (c) their intuitive interpretation of the term 'interior volume'.

Each child was presented with a rectilinear container (a small plastic open rectangular box) and afterwards with a curved container (a small glass vase). For each container, they were asked how they would find its 'volume', how they would find its 'capacity' and what the difference between the 'volume' and 'capacity' of the container was. They were also asked about what they think the 'interior volume' of the container would mean. While holding a container, the child was shown a set of resources: centicubes, play-dough, kinetic sand, a ruler, kitchen scales, balancing scales, water, a measuring jug (in which the containers would fit), and a measuring cylinder.

A combination of scripted and spontaneous probes was used to pursue interesting issues that emerged during the interview (Matteson & Lincoln, 2009). Think-aloud responses were prompted through verbal probes (Matteson & Lincoln, 2009). When a participant spoke softly, the interviewer would repeat their response for clarification. Neutral scripted probes were prepared for whenever the interviewer felt uncertain of the student's intended meaning, as demonstrated in relevant case studies (for example, Curry & Outhred, 2005). If at any time the participants looked confused or were unsure about the meanings of the words ‘volume’ and ‘capacity’, the interviewer offered the syllabus definitions to keep consistency and ensure validity.

Analysis

Data were collected via video recordings in conjunction with field notes recording gestures, hesitations and any salient responses. The video recordings were closely examined by the two researchers who had been present at the interviews and by their supervising lecturer. Salient themes of participants’ responses and understandings were discussed. The video recordings were subsequently transcribed and gestures noted.

Open coding was used to generate categories that helped conceptualise the data (Punch & Oancea, 2014). Quotations and gestures were initially coded as to how each 3D space was understood and how it was measured. It was then realised that the children’s understanding of the space, their measurement strategy and the units used were closely related. Consequently, the data were coded in terms of concepts and collapsed into major themes:

1. Their understanding of the volume of a container.
2. Their understanding of the capacity of a container.
3. Their distinction between volume and capacity.
4. Their interpretation of ‘interior volume’.

The participants’ conceptual changes were also discussed and it was noted whether a conception was held initially, or whether it changed as a result of their request for a definition, engagement with the task and/or the interviewer’s questions. Any discrepancies in interpretation were discussed amongst the researchers until agreement was reached.

Results

1. Understanding of the Volume of a Container

All six children when given the rectilinear container (a box), believed volume to mean the space inside it. Three chose the use of a formula as their first preference for finding its volume but only one of them did this correctly. Drew combined multiplication with division, “I would measure the length...the width...and then the depth...and then divide it by [the depth]”. Emmet used multiplication after lining two sides of the container with centicubes.

Packing the box with centicubes was Chris’ first preference, “you fill it all the way to the top and you see how many is on the bottom...on the side, you count how many rows and times that row by the bottom”. Drew mentioned this could be done “but not, like, precisely”.

When filling was the preferred measurement method for measuring volume, the children thought of capacity. Billy discovers: “… you could fill it with water and...then pour it out into a jug and then see how much water's in it. Oh, that's capacity, oops”. Similarly, when asked about volume, Alex would “get a measuring cup…and see how much water…and that's the capacity”.

When asked whether the volume could be measured in millilitres, five children thought it could - three of them saying that this was because the container “can hold water” (Chris). Billy disagreed “because little cubes and stuff can't be measured in millilitres”.

2. Understanding of the Capacity of a Container

Table 1 shows the children's various interpretations of 'capacity'. Every child described capacity to mean "how much something can hold" and they would all find the capacity of the curved container by filling it with something. However, when asked how they would find the capacity of the box, half of them preferred packing it with centicubes than filling it with water. Questioning these children about capacity either caused confusion, or a realisation that capacity is volume. For the box, Fargo said he preferred to "put cubes in there and fill it up" and Drew recognised the strategy was "similar to volume". Emmet would fill the box "using the blocks," but soon thought, "umm I did what I meant for volume".

Table 1

Conceptions about Capacity held by Alex (A), Billy (B), Chris (C), Drew (D), Emmet (E), and Fargo (F). A shaded box indicates that the child held the conception

Understanding of Capacity	**Student**					
	A	**B**	**C**	**D**	**E**	**F**
Capacity is how much an object/container can hold.	■	■	■	■	■	■
Capacity is about filling.	■	■	■	■	■	■
Capacity can be measured in cubic centimetres.	■		■	■	■	■
Capacity can be about weight.	■		■	■		■

An alternative conception that emerged was that the units, cubic centimetres, could only be used if the centicubes fit without gaps (four children). Cubic centimetres could be used to measure rectilinear containers "because it can hold centicubes easily" (Chris). Drew described cubic centimetres to be "when you're trying to fit cubes into something". Drew, Fargo and Emmet mentioned it cannot be used for a vase as there may be gaps, "There's, like, no umm edges so it'd – you'd have like gaps and stuff so you wouldn't be able to properly measure it" (Fargo).

Capacity was also interpreted as weight. Capacity is "when you're trying to measure how much it weighs," particularly, "at the butcher shop – the food scale" (Chris). It is seen "in constructions, so if they try to build like a, like, an elevator, they can see what's its capacity and how much people it can hold," (Alex).

Four children said they would fill the vase with water and measure its capacity in millilitres. Chris filled the vase with centicubes, weighed the cubes and described the units for capacity as "millilitres, or kilos". Fargo said he would fill it with play-dough to measure capacity and "see, like, how heavy it is…so this could be like thirty grams, so like, the [vase's] capacity would be thirty grams". Drew would weigh the water to measure both the volume and capacity of the container.

3. The Distinction between 'Volume' and 'Capacity'

Volume and capacity were both interpreted as the inside space of the container. All the children except Billy and Drew believed at some stage in the interview, that volume and capacity were similar. Billy thought they were different because volume is about space while capacity is about the substance/material used to fill it. Similarly, Drew said, "capacity is like what you can fit in it and volume is like the room in it".

Chris initially thought that the volume and the capacity of the vase were different because "volume is trying to take up the whole space…and capacity is the space of it, the container or shape, the inside". After further discussion with reference to the container, he said: "wait so, volume and capacity isn't that different". Fargo had earlier thought capacity could be the same as volume as cubes could fill the box so when thinking about the vase he said, "in both ways you're like filling it I guess, so it could be like the same, 'cause you can use cubic centimetres for both of them". He then reverted to thinking volume and capacity are different: "so it's quite different I'd say, because the volume you would be filling it up but you're not like measuring how much you are filling it up".

4. The Interpretation of 'Interior Volume'

All six children interpreted 'interior volume' to mean the space inside, however Chris was prompted with "the hint is in the word" before gesturing the space inside the container. Billy, who saw a clear distinction between volume and capacity, said it was the same volume measured earlier: "last time I was kind of measuring the volume anyway...without saying interior". At this stage of the interview, Alex also made a distinction between capacity and volume saying "capacity is how much things it can hold...interior volume is like *inside* the volume". Two children thought 'interior volume' could also mean capacity. Fargo mentioned that interior volume could be capacity except "with the capacity you're measuring like how much it can hold and then with the volume measuring how much space it takes." Fargo decided that like capacity, interior volume could be measured in millilitres since "you could just like fill it up".

Discussion

The most important finding from this study was that the meaning of 'capacity' is ambiguous and confusing. Every child described capacity as being able to 'hold something' almost word-for-word, perhaps as a result of declarative knowledge, however their measurement methods varied vastly. As stated earlier, capacity can be about holding non-fluid things such as 'people' and it can also be about weight. Some participants chose to weigh the filling material to find the capacity of the container, a possible result of its presence in various contexts outside the mathematics classroom (Meiers & Trevitt, 2010). This confusion may impede an understanding of capacity as volume.

During the interviews, some participants became confused and/or realised that their strategy for finding the volume of a container was identical to finding its capacity. This is possibly a result of learning capacity and volume as if they are separate attributes with separate units, then later attempting to connect the two attributes and their units. When other measurement attributes are 'connected' the situation is somewhat different. Density 'connects' volume with mass. Millilitres 'connect' with grams because one millilitre of water weighs one gram (at 4°C). The connection between one millilitre and one cubic centimetre however, is by definition: one millilitre of any substance or space is *the same as* one cubic centimetre.

Every child *packed* or *filled* containers to measure volume, inferring that the volume of a container is interpreted as its inside space. Volume was interpreted as "how much space it has" whilst capacity as "how much it holds". To distinguish between the volume and capacity of a container they understood volume as an amount of space and capacity as a measure of the material that could fill it. The students' focus on capacity as the filling material may be due to it being measured by transferring the filling (usually water) into another container to determine its volume.

One child believed that millilitres cannot be a unit of volume because cubes, being solid, cannot be measured in millilitres. The syllabus glossary states that capacity "*is used in reference to the volume of fluids or gases and is measured in units such as litres or millilitres*" (ACARA, 2017). The remaining children believed that millilitres could be used as a unit for volume since the container "can hold water". They also believed that cubic centimetres can only be units of capacity when centicubes could neatly fit into the container. Perhaps using a variety of liquid and solid materials, such as play-dough, for filling the whole container might disassociate volume with cube-fitting and capacity with liquid-filling, thereby enabling them to see a millilitre as the same amount of space as a cubic centimetre.

Using a task such as the finding the volume of a plastic rectilinear container and seeking an explanation, may have led to a shift in thinking. As they mostly chose the same filling materials for volume and capacity, it became plausible for capacity to be understood as volume.

All six children interpreted 'interior volume' to mean the space (volume) inside a container. Two children acknowledged that 'interior volume' could also mean capacity. The others, although accepting that volume and capacity were similar, found it hard to agree that they could both be the same 3D space. In their view, the term 'interior volume' described the volume they had measured, but could not describe the capacity of the container because capacity is about *how much is held.* The term 'interior volume' yields a more precise meaning than 'capacity' because it refers only to the space enclosed by a hollow three-dimensional object and it avoids the misconception that capacity is about the filling or the 'fullness' of the container (Harrison, 1987). The term 'interior volume' also clarifies that it is type of volume (3D space) so students are more likely to use fluid units and cubic units interchangeably.

Whether the children thought they could measure the volume of a container in cubic centimetres depended on whether the container was curved or rectilinear. Participants said it was possible to measure the box in cubic centimetres, but not a vase, due to its "round edges". Perhaps this conception is a result of classroom experiences, where students only pack cubes into rectilinear containers, or recreate solids with cubes. This could also be the result of a focus on structural components only in rectilinear containers or objects (Battista, 2003; Outhred et al., 2003).

Conclusion

The purpose of this study was to explore 11 to 12-year-old students' conceptions of the terms of volume, capacity and 'interior volume'. Their responses revealed several alternative conceptions to the standard meanings for these measurement attributes (as defined in the Australian Curriculum). The findings revealed confusion about the relationship between volume and capacity, and about the most appropriate methods for measuring these attributes. Although only six children were interviewed, they came from six different schools, suggesting that the children's responses were not the result of particular learning experiences. The children's consistent response to the term 'interior volume' indicates the potential of using it as an alternative to 'capacity', to minimize confusion and maintain the conceptual focus on volume as an amount of 3D space. Further research is needed to more thoroughly explore this hypothesis.

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